A method for Sequential Thinning of Digital Raster Terrain Models II: mixed locally adaptive wavelet-splines and anisotropy

OLGA WÄLDER, Dresden

Keywords: digital raster terrain models, DTM, mathematical method for sequential thinning of DTM, wavelet concept

Zusammenfassung: *Ein Verfahren zur sequentiellen Ausdünnung von digitalen Raster Terrain Modellen*. Diese Arbeit ist eine ergänzende Fortsetzung der vorherigen Veröffentlichung WÄLDER & BUCHROITHNER (2004), PFG Heft 3/2004, S. 215–220, zur sequentiellen Ausdünnung bzw. zur funktionalen Geländeapproximation mittels spezieller „Wellen-Strukturen“. Nun wird für das reale Georelief der Aufbau von Makro- bis Mikrostrukturen so organisiert, dass nicht nur die Amplituden, sondern auch die Form der Wavelet-Spline-Komponenten optimal angepasst werden. Außerdem kann das anisotrope Georelief speziell behandelt werden, so dass seine Geländeapproximation mittels Wavelet-Splines möglichst „naturnahe“ durchgeführt wird. Das verallgemeinerte Verfahren wird zur optimalen Gegenüberstellung anhand eines digitalen Geländemodells von einem Testgebiet in den österreichischen Alpen demonstriert, das schon in der vorherigen Veröffentlichung untersucht wurde. Nähere Details zu diesem verallgemeinerten Ansatz werden diskutiert.

Summary: This paper presents a complementary continuation of the previous paper WÄLDER & BUCHROITHNER (2004) about a sequential method for the thinning and the description of digital raster terrain models based on wavelet concepts by means of approximating functions. Now, the composition from macrostructures to microstructures of the geo-relief is organized, that the kind of functions in a wavelet-spline can be optimally fitted in addition to the amplitudes. Furthermore, the anisotropic geo-relief can be treated in a special way, that its approximation should be “natural” as far as possible. The generalized method is demonstrated using the DTM of the test area in the Austrian Alps, that was treated in the previous paper with the aim of better comparision. Further details to this refined approach are discussed.

1 Modelling with mixed locally adaptive wavelet-splines: a short overview

The developed methods and algorithms for the structuring and thinning of the relief data with wavelet/splines were presented and tested for real geo-relief data in WÄLDER & BUCHROITHNER (2004). The classical wavelet theory is well-known and often applied in signal processing, physics, photogrammetry, cartography a.o., see FABER (2004), MEIER (2003), SCHMIDT (2001), TING JIANG (1997). Our method differs from usual approaches because of its locally adaptivity and flexibility in the calculation of amplitudes, see WÄLDER & BUCHROITHNER (2004). Here, we only give a short overview about the preceding modelling.

It is assumed, that the unknown function $z = z(x, y)$ allows the following sequential approximation (1), c.f. WÄLDER & BUCHROITHNER (2004):

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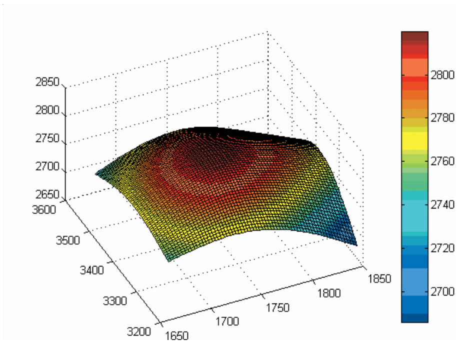

Fig. 1a:

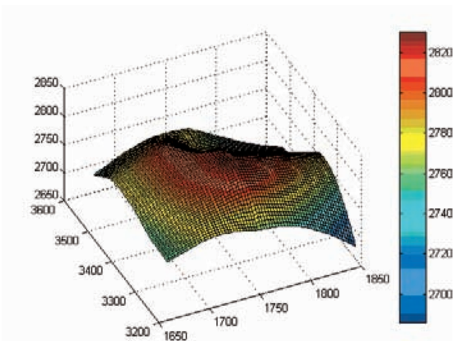

Fig. 1b:

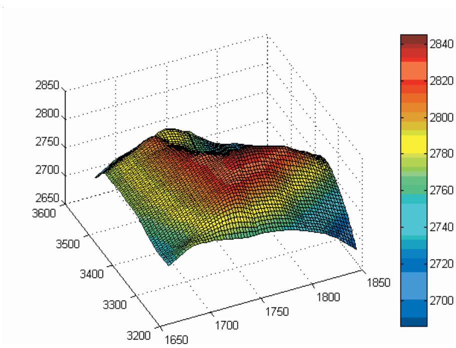

Fig. 1c:

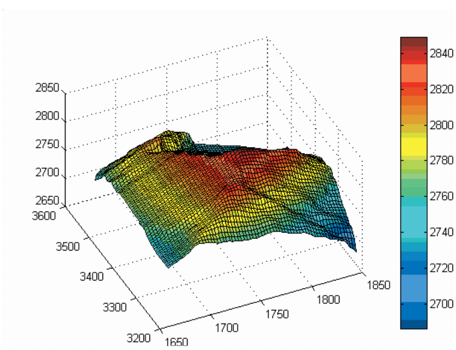

Fig. 1d:

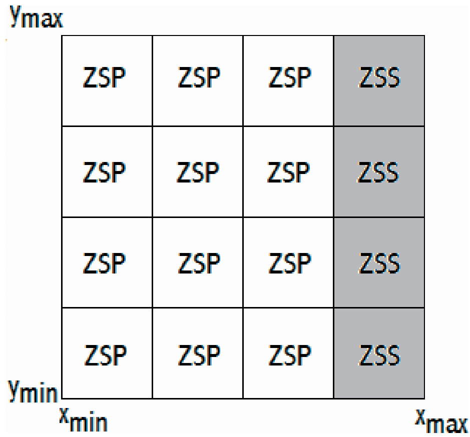

Fig. 1e: The combinations of wavelets after step 3: Z is a zigzag-like wavelet, S is a sinusoidal wavelet, P is a polinomial wavelet.

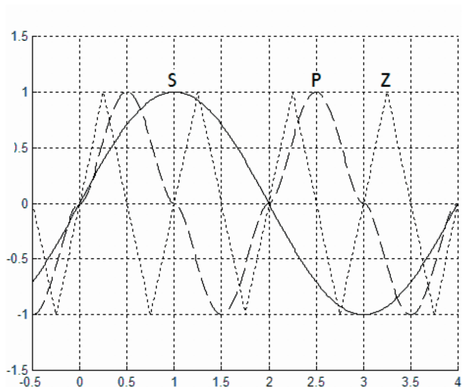

Fig. 1f: Three different kinds of wavelets: S is a sinusoidal wavelet $f_1(\cdot)$, P is a polinomial wavelet $f_2(\cdot)$, Z is a zigzag-like wavelet $f_3(\cdot)$, c.f. (1).

Fig. 1a–d: Approximation of the thinned DTM with a mixed locally adaptive wavelet-spline after: a) step 2, b) step 3, c) step 4, d) step 7.

$$P_1(x,y) = a_0 + a_1 f_1(x) + b_1 f_1(y) + c_1 f_1(x) f_1(y),$$
$$P_2(x,y) = P_1(x,y) + a_2(x,y) f_2(x) + b_2(x,y) f_2(y) + c_2(x,y) f_2(x) f_1(y), \ldots$$
$$P_{k+1}(x,y) = P_k(x,y) + a_{k+1}(x,y) f_{k+1}(x) + b_{k+1}(x,y) f_{k+1}(y) + c_{k+1}(x,y) f_{k+1}(x) f_{k+1}(y), \ldots$$
$$k = 0, 1, \ldots, \quad (1)$$

where $f_k(\cdot)$, $k = 1, 2, 3, \ldots$ are special wavelets. Their possible kinds are sinusoidal, zigzag-like, polynomial ect., see Fig. 1f. For example, for sinusoidal wavelets these functions are:

$$f_k(x) = \sin(2^{k-2}\pi x), \ k = 1, 2, \ldots \quad (2)$$

The coefficients a, b, c are amplitudes. These constants are individual for each cell. This means, that they can vary for different cells. Exactly this leads to the *locally adaptive* wavelet-splines in comparison to the classical wavelet approaches.

But not only the amplitudes are individualy chosen: the kind of functions $f_k(x)$, $k = 1, 2, \ldots$ in (1) can be step-wise changed. Thus, these locally adaptive wavelets are called *mixed* in addition. With the aim of choosing the kind of functions the different combinations should be a-priori produced and compared. Then, this combination is finally accepted, that leads to the exactest approximation of the thinned DTM.

Fig. 1a–c shows three steps (from step 2 to step 4) of spline-modelling with mixed wavelet-splines for the test area. The locally adaptive combinations of wavelets after step 3 can be seen in Fig. 1e. Fig. 1f shows three different kinds of used wavelets. Fig. 1d presets the final, seventh step of the approximation. Tab. 1 presents the comparison between different kinds of wavelet-spline-approximations.

Tab. 1: Accuracies in [m] obtained by different wavelet-splines. The thinned DTM corresponds to the seventh step.

Granat Spitz test area

Spine kind	Step 1	2	3	4	5	6
Sinusoidal (single)	69.30	15.43	5.18	2.17	1.02	0.39
Zigzag-like (single)	62.03	20.07	7.01	2.27	1.03	0.39
Polynomial (single)	63.87	17.91	6.97	2.72	1.22	0.44
Mixed	62.03	15.26	4.94	1.97	1.00	0.37

2 Modelling the anisotropy

At first, we introduce some definitions, which we use for modelling the anisotropy.

Definition 1: A closed set P^2 from R^2 is called *X-convex*, if for

$$\forall \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}, \begin{pmatrix} x \\ y_0 \end{pmatrix} \in P^2 \text{ and } \forall a \in [0,1], \text{ it yields}$$

$$v_x = a \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + (1-a) \begin{pmatrix} x \\ y_0 \end{pmatrix} \in P^2.$$

Remark: This definition is weaker as the classical definition of a convex set.

Definition 2: A transformation of a $N \times N$-grid from a X-convex set to $N \times N$ – grid with (x, y)-coordinats from $(0,1) \times (0,1)$ is called *generalized $(0,1) \times (0,1)$-transformation*, if:

$$x_j^{(0,1)}(y_i) = \frac{x_j - \min(x(y_i))}{\max(x(y_i)) - \min(x(y_i))}, \quad (3)$$
$$y_i^{(0,1)} = \frac{y_i - \min(y)}{\max(y) - \min(y)},$$

and its re-transformation is:

$$x_j = \{\max(x(y_i)) - \min(x(y_i))\} x_j^{(0,1)}(y_i) + \min(x(y_i)),$$
$$y_i = \{\max(y) - \min(y)\} y_i^{(0,1)} + \min(y),$$
$$i, j = 1, \ldots, N.$$

Assumption: We assume, that the test area can be divided into a finite number of X-convexe, isotrop subsets. Then, we use definition 2 and calculate separately mixed wavelet-splines for each subset.

Remark: Definitions 1 and 2 can be simply adopted for Y-convexity. For example, the test area can be covered with Voronoi cells, which are X-convex as well as Y-convex. Thereby, the crossings between locally iso-

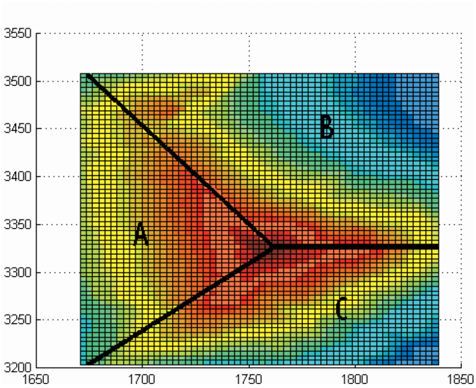

Fig. 2: Subsets of the Granat Spitz test area, which are assumed to be isotrop.

Tab. 2: Accuracies in [m] obtained for three subsets using mixed locally adaptive wavelet-splines. The thinned DTMs corresponds to the seventh step.

Granat Spitz test area, subsets A-C						
Step Subset	1	2	3	4	5	6
A	38.26	7.63	4.21	1.51	1.07	0.47
B	11.00	7.41	3.14	1.37	0.61	0.27
C	21.83	4.86	3.25	1.59	0.72	0.32

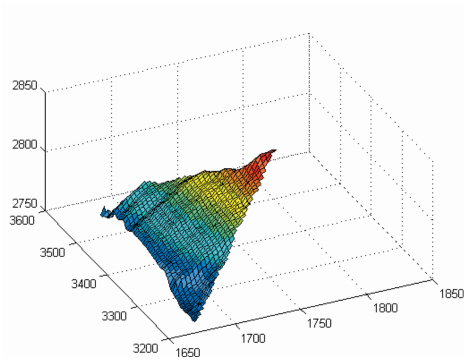

Fig. 3a:

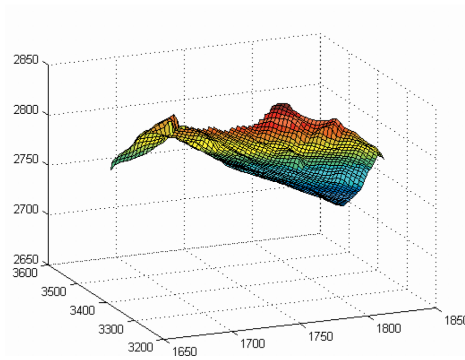

Fig. 3b:

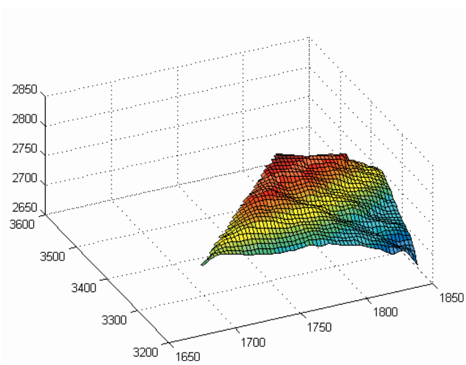

Fig. 3c:

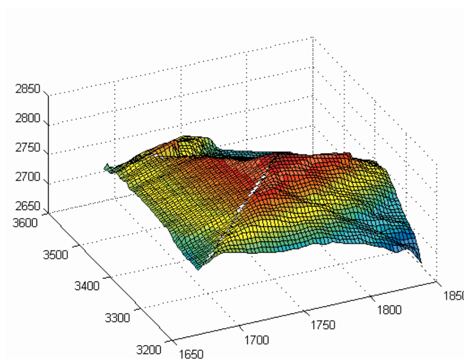

Fig. 3d:

Fig. 3a–d: Approximation of the thinned DTM with mixed locally adaptive wavelet-splines for:

a) subset A of the Granat Spitz test area,
b) subset B of the Granat Spitz test area,
c) subset C of the Granat Spitz test area.
d) The whole Granat Spitz test area combined from subsets a)–c). The omitted z-connections should underline the anisotropy of the test area.

trop subsets of the anisotrop test area can be used as natural edges of these Voronoi cells.

In section 3 we show an application of this approach. The assumption made above leads to the use of mixed wavelet-splines. Their re-transformated frequences are locally adapted corresponding to the y-coordinate.

3 Case Study Austrian Alps (Granatspitz Massif)

The test area belongs to the Austrian Hohe Tauern, southwest of Zell am See. This high-alpine area reaches from approx. 800 m to 3150 m a.s.l. This test area is characterized by a richness in geomorphological forms and therefore perfectly suitable as a test area for mixed locally adaptive wavelet-splines.

Tab. 1 presents the comparison of the accuracies obtained by single and mixed wavelet-splines. The thinned DTM is here a 65×65 grid with 64×64 different cells. Twenty two different amplitudes and seven combinations of kinds of wavelets in the mixed locally adaptive wavelet-spline belong to each cell.

The splining or, with other words, the approximation of the relief with mixed locally adaptive wavelet-splines is obviously more precise and less smoothed in comparison with another approximations of the relief with single locally adaptive wavelet-splines, cf. Tab. 1. Thus, it seems to be better suitable for modelling real reliefs. On the other hand, we should save the additional information about the combinations of these kinds for each cell. Fig. 1e shows these optimal combinations after step 3.

In order to illustrate the modelling of the anisotropy, we firstly divide the test area into three subsets, which are assumed to be nearly isotrop, see Fig. 2. At second, we choose the 7-step-thinning of these sets and the splining of the obtained grids with the mixed wavelet-spline. So, we obtain three new, 65×65 grids with 64×64 different cells after this procedure. The accuracies obtained thereby can be found in Tab. 2. Fig. 3a–c present the 7-step-approximations of the subsets, Fig. 3d shows the whole approximation of the test area. Here, the crossings between the subsets are special underlined by omitted z-connections.

4 Discussion

Statistical modelling of a relief can help to describe the actual geomorphological features and to understand it at different levels of detail. The approach, presented in Wälder & Buchroithner (2004), is generalized by introducing different kinds of functions in (1) as well as by considering anisotropic cases. This approach is applied for the Granat Spitz test area. The approach to use locally adaptive mixed wavelet-splines and transformation (3) leads to flexibility in the choice of the kind of wavelets as well as of their frequences. So, small subsets are covered by wavelets with higher frequences as large subsets for the same step of the approximation.

All procedures are realized in the programming language Delphi 6.0. MATLAB is used for visualizations.

The relief modelling can be further refined for two or more time epochs by using surface deformation analysis between these epochs. First investigations have been started for a glacier region in the Oetztal Alps.

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Address of the author:

Dr. rer. nat. Olga Wälder
Institute for Cartography
Dresden University of Technology
Mommsenstrasse 13, D-01062 Dresden
Tel.: +49-351-463-3-6200
Fax: +49-351-463-3-7028
e-mail: Olga.Waelder@mailbox.tu-dresden.de

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