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Transformation of Dynamic Systems Describing Dynamic Objects

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Summary: Transformations of Cartesian coordinate systems are considered in order to use them for geodetic tyings of dynamic objects. A definition of geodetic monitoring of dynamic objects is given. The main attention is devoted to continuous rotations and instantaneous turns. Direct and inverse algorithms of transformations from local to Greenwich and inertial systems are considered.

Zusammenfassung: *Transformation dynamischer Systeme zur Beschreibung dynamischer Objekte*. Betrachtungen zur Transformation Cartesischer Koordinatensysteme für die Berechnung geodätischer Objekte. Eine Definition zur geodätischen Erfassung dynamischer Objekte wird dargestellt. Schwerpunkte der Betrachtungen sind die kontinuierliche Rotation und momentane Drehungen. Betrachtet werden direkte und inverse Algorithmen zur Transformation von lokalen Systemen ins Greenwich-System und inertiale Systeme.

1 Definition and statement of problems

Dynamic systems will be considered as rectangular (Cartesian) coordinate systems executing translational and rotary motions. The objects situated in these systems may themselves have translational and rotary motions.

Definition: Geodetic monitoring will be considered to consist in periodic observations of dynamic systems and objects, that ensure a preset accuracy σ_n within an interval $[t_0, t]$ in the definition of the equations of motions of systems and objects.

In the present paper we will consider translational motions that are linear

$$\Delta\bar{r} = \Delta\bar{r}_0 + \Delta\dot{\bar{r}}_0(t - t_0) \tag{1}$$

or nonlinear

$$\Delta\bar{r} = \Delta\bar{r}_0 + \Delta\dot{\bar{r}}_0(t - t_0) + \frac{\Delta\ddot{\bar{r}}_0}{2}(t - t_0)^2, \quad (\Delta\dddot{\bar{r}}_0 = 0) \tag{2}$$

Notice that transformations of Cartesian, rectangular systems are, in case of translational motions, carried out by trivial parallel translation. As for the dynamics of the objects to be considered, the objects are generally to be represented by material points and hence, only a translational motion will be taken into account. Parallel translation and rotation of the system are carried out by means of the well-known equation [1]:

$$\bar{r} = \Delta\bar{r} + \mu \cdot \Pi \cdot \bar{R} \tag{3}$$

where $\bar{r}$ and $\bar{R}$ are radius vectors locating the objects in the new and old systems, $\Delta\bar{r}$ is the radius vector of parallel translation of the old system to the new one, coefficient μ is a scale factor, Π is a matrix of order 3 that consists of direction cosines, which may be constant or have a weak or strong dependence on time t. On the assumption that the scale factors of the old and new systems equal, $\mu = 1$.

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Let us consider three cases:

1. In the old system $\bar{R}$ is motionless, and matrix $\Pi(t)$ and $\Delta\bar{r}(t)$ are functions of time t;
2. In the old system, with objects $\bar{R}(t)$ moving, matrix Π is independent of time t, and we will suppose that $\Delta\bar{r} = 0$;
3. In the old system, with objects $\bar{R}(t)$ moving, the origin and orientation of the system are variables, i. e. $\bar{R}(t)$, $\Pi(t)$ and $\Delta\bar{r}(t)$ are functions of time t.

In all cases, in addition to the transformations of the systems and objects, we have to transform their velocities and accelerations.

2 The First Case of Transformation

We take the first and second derivations with respect to t using equation (3) and taking into account the first case and equation (2):

$$\begin{aligned} \dot{\bar{r}} &= \Delta\dot{\bar{r}}_0 + \Delta\ddot{\bar{r}}_0(t - t_0) + \dot{\Pi}\bar{R}\,, \\ \ddot{\bar{r}} &= \Delta\ddot{\bar{r}}_0 + \ddot{\Pi}\bar{R}\,, \end{aligned} \tag{4}$$

where we suppose that changes of acceleration of the origin of system $\bar{R}$ are absent. Taking into account that matrix Π can always be presented in exponential form [4], i. e. $\Pi = e^C$, where C is a skew-symmetric matrix of the direction cosines of the axis of rotation in system $\bar{R}$ [1], we have

$$\begin{aligned} \bar{r} &= \Delta\bar{r} + e^C\bar{R}\,, \\ \dot{\bar{r}} &= \Delta\dot{\bar{r}}_0 + \Delta\ddot{\bar{r}}_0(t - t_0) + \dot{C}\cdot(\bar{r} - \Delta\bar{r})\,, \\ \ddot{\bar{r}} &= \Delta\ddot{\bar{r}}_0 + (\ddot{C} + \dot{C}^2)\cdot(\bar{r} - \Delta\bar{r})\,, \end{aligned} \tag{5}$$

from which we substituted the expression $e^C\bar{R}$ using the first equation in (5).

Matrix C has the next form:

$$C = \begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix} \tag{6}$$

where $c_1 = \dfrac{n_{32} - n_{23}}{2\sin\delta}$, $c_2 = \dfrac{n_{31} - n_{13}}{2\sin\delta}$,

and $c_3 = \dfrac{n_{21} - n_{12}}{2\sin\delta}$. (7)

Let us coin the following terms:

$$c_1^0 = (n_{32} - n_{23})/2, \quad c_2^0 = (n_{31} - n_{13})/2, \quad c_3^0 = (n_{21} - n_{12})/2, \tag{8}$$

where n_{ij} are non-diagonal elements of matrix Π.

Now, let us return to skew-symmetric matrix C, which will be to get the next form bering in mind (6)–(8):

$$C = \begin{pmatrix} 0 & -c_3^0 & c_2^0 \\ c_3^0 & 0 & -c_1^0 \\ -c_2^0 & c_1^0 & 0 \end{pmatrix} \cdot \frac{1}{\sin\delta} = C^0/\sin\delta \tag{9}$$

Taking the derivative of expression (9) with respect to t, we have

$$\dot{C} = (\dot{C}^0/\sin\delta - C\cdot\cos\delta\cdot\dot{\delta})/\sin\delta \tag{10}$$

The first summand of the expression (10) determines the axis of rotation, changing its position in space. A turn around this axis is carried out by means of an instanteneous turn, i. e. angle δ in the first summand does not depends on time t. From (7) and (8) it is easy to see that motion and fixation of the axis of rotation provides non-diagonal elements of matrix Π.

But as the instantaneous turn of angle δ will look like this [2]

$$\delta = \arccos\frac{n_{11} + n_{22} + n_{33} - 1}{2} \tag{11}$$

the value of the angle of turn δ is defined by means of diagonal elements of matrix Π. Note that the second summand in (10) carries out the turn through an infinitesimal angle $d\delta$, so we can interpret these expressions as projections of the instantaneous rate of rotation, and if parameter t is time, the second summand in (10) given above are projections of the angular rate of rotation, i. e.

$$c_n ctg\delta\cdot d\delta = \omega_n \tag{12}$$

and $c_n ctg\delta\cdot d\delta$ are projections of the angular rate $\bar{\omega}$ on axes Ox, Oy and Oz [3].

Therefore let us rewrite the second summand (9) as [2]

$$\dot{C}_2 = \begin{pmatrix} 0 & -\omega_Z & \omega_Y \\ \omega_Z & 0 & -\omega_X \\ -\omega_Y & \omega_X & 0 \end{pmatrix} \tag{13}$$

The second derivative from expession (10) with respect to t will look like this

$$\ddot{C} = \frac{d}{dt}(\dot{C}^0/\sin^2\delta - \dot{C}_2) \tag{14}$$

Differentiated with respect to time t of (14) we can get in explicit form the expression for $\ddot{C}$.

3 The Second Case of Transformation

This case describes the transformation of fixed systems in which objects are in motion. Let us address some specific problems. Let an object M be in motion relative to the Cartesian system of coordinates called "local" [3]. The local system is given in the following manner: The origin of system is located at point P, which is motionless in relation to the terrestrial surface. Axis PZ is directed at the nadir point, axis PX is directed towards the North Pole as a tangent to the meridian, and axis PY is directed to a point complementing the right-handed system. In the local system described above, let us give the object M mentioned above a radius vector $\bar{R}_l(X_l, Y_l, Z_l)$, a velocity $\dot{\bar{R}}_l(\dot{X}_l, \dot{Y}_l, \dot{Z}_l)$ and an acceleration $\ddot{\bar{R}}_l(\ddot{X}_l, \ddot{Y}_l, \ddot{Z}_l)$. The orientation of the local system at the basic point P is fixed in relation to the Greenwich system of coordinates.

Therefore, the coordinates, velocities and accelerations will be transformed to the Greenwich system by the same matrix Π_{El}, i. e.

$$\bar{R}_E = \Pi_{El}\cdot\bar{R}_l, \quad \dot{\bar{R}}_E = \Pi_{El}\cdot\dot{\bar{R}}_l,$$
$$\ddot{\bar{R}}_E = \Pi_{El}\cdot\ddot{\bar{R}}_l. \tag{15}$$

A notation for the generalized column vector of order 9 is coined by

$$\bar{Q} = (X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}, \ddot{X}, \ddot{Y}, \ddot{Z})^T \tag{16}$$

with the appropriate indices (E) or (l) for the Greenwich and local systems. Above the generalized matrix Π_{El} we have

$$\Xi_{El} = \begin{pmatrix} \Pi_{El} & 0 & 0 \\ 0 & \Pi_{El} & 0 \\ 0 & 0 & \Pi_{El} \end{pmatrix} \tag{17}$$

where Ξ_{El} is a hypermatrix, and the 0s are zero matrices of order 3×3; the content of a matrix Π_{El} will be considered below. For the transformation from the local to the Greenwich system we obtain the following expression

$$\bar{Q}_E = \Xi_{El}\cdot\bar{Q}_l \tag{18}$$

which will simultaneously transform coordinates, velocities and accelerations.

In Fig. 1, $X_E O Y_E Z_E$ is the Greenwich system of coordinates with the origin at the centre of the mass of the Earth.

$X_l P Y_l Z_l$ is a local Cartesian coordinate system. The orientation of this system is determined above.

Let object M be given in the polar coordinate system:

ρ is the polar distance,
A is the azimuth of a direction PM,
z is the zenith distance (or height h = 90 − z) of a direction PM

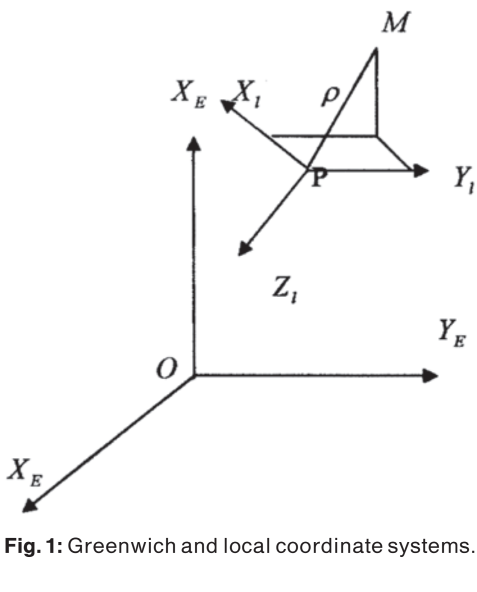

Fig. 1: Greenwich and local coordinate systems.

The dependence between the Cartesian and polar coordinates has the form:

$$\bar{R}_l = \begin{pmatrix} X_l \\ Y_l \\ Z_l \end{pmatrix} = \rho \begin{pmatrix} \cos(360 - A)\sin(180 - z \\ \sin(360 - A)\sin(180 - z) \\ \cos(180 - z) \end{pmatrix} \tag{19}$$

There remains to be defined the expression for a matrix Π_{El}. Taking Euler's angles, which are standard in space geodesy and celestial mechanics, we have $\Omega = 180 - \lambda$, $i = 90 + \varphi$, and $\omega = 0$, where φ and λ are the astronomical coordinates of point P. In this case, matrix Π will have the following form

$$\begin{aligned} \Pi_{El} &= \Pi_{OZ_E}(180+\lambda)\cdot\Pi_{OY_E}(90+\varphi)\cdot\Pi_{OZ_E}(0) \\ &= \begin{pmatrix} -\cos\lambda & \sin\lambda & 0 \\ -\sin\lambda & -\cos\lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &\cdot \begin{pmatrix} -\sin\varphi & 0 & -\cos\varphi \\ 0 & 1 & 0 \\ \cos\varphi & 0 & -\sin\varphi \end{pmatrix} \end{aligned} \tag{20}$$

The final expression for matrix Π_{El} will be given below, but we suppose that expressions (20) are suitable for use, as in any environment (Mathcad, Mathematika, MatLab, Maple etc.) it is easier to write down the expression in this form, and multiplication of matrixes is performed automatically. Nevertheless, we have

$$\Pi_{El} = \begin{pmatrix} \sin\varphi\cos\lambda & \sin\lambda & \cos\varphi\cos\lambda \\ \sin\varphi\sin\lambda & -\cos\lambda & \cos\varphi\sin\lambda \\ \cos\varphi & 0 & -\sin\varphi \end{pmatrix} \tag{21}$$

The transformations on the basis of astronomical coordinates will result in low accuracy, but such tying is independent. Where a high accuracy of tying is essential, then the coordinates of point P should be known in the national reference frame system (CK-42 etc.) or in one determined in the satellite Greenwich system (WGS-84 or ПЗ-90 etc.). The connection between different reference systems is given by means of known equations of rotation and parallel translation of coordinate systems and the scale factor. The main value of such an equation (for example (3)) is represented by the values of numerical constants that are included in this equation.

Transition from the local geodetic into the Greenwich system of coordinates, velocities and accelerations will be executed on the basis of eq. (18), but it is necessary to formally replace astronomical coordinates φ and λ by the geodetic ones B and L in a matrix Π_{El} with expressions (20) or (21).

4 The Third Case of Transformation

Let us consider the third case, in which the coordinates of the points and the origin of a local system are variable. Suppose they are changed continuously or instantaneously. We take the first and second derivatives with respect to t of equation (3), taking (2) into account:

$$\begin{aligned} \dot{\bar{r}} &= \Delta\dot{\bar{r}}_0 + \Delta\ddot{\bar{r}}_0(t - t) + \dot{\Pi}\bar{R} + \Pi\dot{\bar{R}} \\ \ddot{\bar{r}} &= \Delta\ddot{\bar{r}}_0 + \ddot{\Pi}\bar{R} + 2\dot{\Pi}\dot{\bar{R}} + \Pi\ddot{\bar{R}} \end{aligned} \tag{22}$$

As shown in [3], the equations for instantaneous and continuous rotations are invariable. Below we will consider the more general case of continuous rotations. By equation (16) above we have introduced a generalized column vector of order 9-$\bar{Q}_{El}$, consisting of coordinates, velocities and accelerations. As for the generalized matrix of transformation, it has the same order (9 on 9), but H_{El} differs from a matrix Ξ_{El} by its contents.

The hypermatrix Π of this case will look like this:

$$\mathrm{H}_{El} = \begin{pmatrix} \Pi_{El} & 0 & 0 \\ \dot{\Pi}_{El} & \Pi_{El} & 0 \\ \ddot{\Pi}_{El} & 2\dot{\Pi}_{El} & \Pi_{El} \end{pmatrix} \tag{23}$$

where Π_{El} has the form of (20) or (21), and there are zero matrices of order 3 on 3. As to the derivative with respect to t from a matrix Π_{El}, we shall designate the rotation around an axis OZ by means of an index 3, and around an axis OY by means of an index 2. We have

$$\dot{\Pi} = \dot{\Pi}_3\Pi_2 + \Pi_3\dot{\Pi}_2,$$
$$\ddot{\Pi} = \ddot{\Pi}_3\Pi_2 + 2\,\dot{\Pi}_3\dot{\Pi}_2 + \Pi_3\ddot{\Pi}_2, \tag{24}$$

in which, based on (20), we obtain the derivatives by means of the differentiation with respect to t of the trigonometrical functions of sine and cosine. For example, the expression for $\dot{\Pi}$ can be written as

$$\dot{\Pi} = \begin{pmatrix} \sin\lambda & \cos\lambda & 0 \\ -\cos\lambda & \sin\lambda & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \Pi_2 \cdot \dot{\lambda} + \Pi_3 \cdot \begin{pmatrix} -\cos\varphi & 0 & \sin\varphi \\ 0 & 0 & 0 \\ -\sin\varphi & 0 & -\cos\varphi \end{pmatrix} \cdot \dot{\varphi} \tag{25}$$

where $\Pi_2(\varphi)$ and $\mathit{\Pi}_3(\lambda)$ are given by equation (20) in an obvious form. Similarly for $\ddot{\Pi}$ we have an expression of the form

$$\ddot{\Pi} = \begin{pmatrix} \cos\lambda & -\sin\lambda & 0 \\ \sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \Pi_2 \cdot \ddot{\lambda} + 2\,\dot{\Pi}_3\dot{\Pi}_2 \cdot \dot{\lambda} \cdot \dot{\varphi} + \Pi_3 \cdot \begin{pmatrix} \sin\varphi & 0 & \cos\varphi \\ 0 & 0 & 0 \\ -\cos\varphi & 0 & \sin\varphi \end{pmatrix} \cdot \ddot{\varphi} \tag{26}$$

where $\dot{\Pi}_3(\lambda)$ and $\dot{\Pi}_2(\varphi)$ are available from equation (25) in an obvious form.

As to the expressions for $\dot{\lambda}$, $\dot{\varphi}$, $\ddot{\lambda}u\ddot{\varphi}$, it is necessary to know the dependence of $\lambda u\varphi$ on time t. The linear dependence (according to (2))

$$\begin{pmatrix} \lambda \\ \varphi \end{pmatrix} = \begin{pmatrix} \lambda_0 \\ \varphi_0 \end{pmatrix} + \begin{pmatrix} a_0 \\ e_0 \end{pmatrix} (t - t_0)$$

yields the expressions $\dot{\lambda} = a_0 u\dot{\varphi} = e_0$ and $\ddot{\lambda} = 0u\ddot{\varphi} = 0$.

For a nonlinear dependence, the equation will be similar to expressions in (2). Final transformations will look like this:

$$\bar{Q}_E = \Delta Q_E + H_{El}\bar{Q}_l \tag{27}$$

where $\bar{Q}$ is represented by expression (16), H_{El} is represented by matrix (23), and H is represented by the equations (20), (21) and (24)–(26), i. e.

$$\Delta Q_E = \begin{pmatrix} \Delta\bar{r}_0 & \Delta\dot{\bar{r}}_0 & \Delta\ddot{\bar{r}}_0 \\ \Delta\dot{\bar{r}}_0 & \Delta\ddot{\bar{r}}_0 & 0 \\ \Delta\ddot{\bar{r}}_0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ (t - t_0) \\ \dfrac{(t - t_0)^2}{2!} \end{pmatrix} \tag{28}$$

where the first matrix in (28) has the order (3 on 9).

Thus, equations (18)–(28) solve, by classical methods of geodesy and astronomy, the problem of the geodetic tyings of mobile and motionless objects that are situated either on a terrestrial surface or in the atmosphere or in the near space, as well as that of transforming such objects to a terrestrial geodetic system of coordinates by means of the universal algorithm. The inverse transformation is carried out by means of the transposed matrices Π_{El}, $\dot{\Pi}_{El}$ and $\ddot{\Pi}_{El}$ of matrix H_{El} (23), and we can replace the old coordinates with new ones.

5 Transformation of a Terrestrial System into an Inertial One

Finally, we will consider the following case. Let us suppose that GPS or GLONASS receivers are installed on the object M considered above, and that there is the possibility to receive coordinates of object M in a geodetic system (WGS-84 or ПЗ-90). The measurements will be considered in a dynamic mode, and in this mode we carry out a precise registration of time t.

In this case we can determine coordinates, velocities and accelerations of object M, i. e.

$$X_{ME},\ Y_{ME},\ Z_{ME},\ \dot{X}_{ME},\ \dot{Y}_{ME},\ \dot{Z}_{ME},$$
$$\ddot{X}_{ME},\ \ddot{Y}_{ME},\ \ddot{Z}_{ME}.$$

In this case there is no need to use the algorithms described above, as the coordinates of object M are received directly in the Greenwich geodetic system of coordinates. It is necessary, though, to consider a problem of transformation from the Greenwich geodetic system to an inertial system of coordinates (ICRS). For object M, this connection for will look like this [4]:

$$\bar{r}_{Mi} = (P \cdot S \cdot N \cdot \Pi)_{iE} \cdot \bar{R}_{ME}, \quad (29)$$

where P is the matrix of transition from instantaneous coordinates to average coordinates (accounting for the movement of poles in case of a terrestrial surface), S is the matrix which takes into account the rotation of the Earth (a matrix of sidereal time), N is the matrix of nutation, and Π is the matrix of precession. An obvious form of the matrices listed is present in [5] or in any textbook on space geodesy, spherical astronomy or astrometry. Let us introduce a designation

$$\mathrm{H}_{iE} = (P \cdot S \cdot N \cdot M)_{iE} \quad (30)$$

Then, for expression (29) and accounting for (30), we take the first and second derivatives and thus obtain the following equations:

$$\begin{aligned} \bar{r}_{iM} &= \mathrm{H}_{iE}\bar{R}_{EM}, \\ \dot{\bar{r}}_{iM} &= \dot{\mathrm{H}}_{iE}\bar{R}_{EM} + \mathrm{H}_{iE}\dot{\bar{R}}_{EM}, \\ \ddot{\bar{r}}_{iM} &= \ddot{\mathrm{H}}_{iE}\bar{R}_{EM} + 2\,\dot{\mathrm{H}}_{iM}\dot{\bar{R}}_{EM} + \mathrm{H}_{iE}\ddot{\bar{R}}_{EM}. \end{aligned} \quad (31)$$

In view of the above generalizations, transformations from the Greenwich system to an inertial system can be presented as

$$\bar{q}_{iM} = \Xi_{iE}\bar{Q}_{EM}, \quad (32)$$

where $\bar{q}_{iM} = (x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}\ \ddot{x}\ \ddot{y}\ \ddot{z})^T_{iM}$ $\bar{Q}_{EM}$ looks like (16), and the matrix Ξ_{iM} looks like matrix (23), i. e.

$$\Xi_{iE} = \begin{pmatrix} \mathrm{H} & 0 & 0 \\ \dot{\mathrm{H}} & \mathrm{H} & 0 \\ \ddot{\mathrm{H}} & 2\dot{\mathrm{H}} & \mathrm{H} \end{pmatrix}_{iE} \quad (33)$$

where Ξ_{iE} is the hypermatrix. Each element of this matrix in turn represents matrices of order 3 by 3.

Let us consider the structure of a matrix H_{iE}. All matrices (except S) included as components of (30) depend only weakly on current time t. In fact, these changes do not exceed

1″ arcs per year for a matrix P,
10″ arcs per year for a matrix N,
1′ arcs per year for a matrix M.

In geodetic and astronomical practice, these changes take into account intervals of time that considerably exceed one year; besides, they are also taken into account as instantaneous. As for matrix S, although it carries out a continuous rotation, it is taken into account as an instantaneous turn through a value of sidereal time s, in order to unify the transformation. It concerns the transformation of coordinates, i. e. the first row of hypermatrix Ξ_{iE}.

As to the transformation of velocities and accelerations we are compelled to carry out a continuous rotation, since the angular velocity of the rotation of the Earth enters the matrices of transformation in an obvious form. Supposing that only matrix S depends from time t, we have

$$\begin{aligned} \dot{\mathrm{H}}_{\mathrm{iE}} &= (P\dot{S}NM)_{iE} = (PSNM\dot{S}S^T)_{iE} \\ &= (H\Omega)_{iE}, \end{aligned} \quad (34)$$

where the matrix Ω is a matrix of the Earth's angular velocity $\bar{\omega}$. As the axis of rotation of the Earth, both in inertial and Greenwich systems, actually coincides with axes oz and oZ, matrix Ω looks like this:

$$\Omega_{iE} \approx \begin{pmatrix} 0 & -\omega_z & 0 \\ \omega_z & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

or

$$\Omega_{iE} = \begin{pmatrix} 0 & -\omega_Z & \Delta\omega_Y \\ \omega_Z & 0 & -\Delta\omega_X \\ -\Delta\omega_Y & \Delta\omega_X & 0 \end{pmatrix} \quad (35)$$

Similarly, for acceleration transformation we have

$$\ddot{\mathrm{H}}_{iE} = H_{iE}\dot{\Omega}_{iE} \quad (36)$$

Accounting for (34) and (36), the hypermatrix Ξ_{iE} in expression (33) can be represented as

$$\Xi_{iE} = \mathrm{H}_{\mathrm{iE}} \begin{pmatrix} I & 0 & 0 \\ \Omega & I & 0 \\ \dot{\Omega} & 2\Omega & I \end{pmatrix}_{iE} \quad (37)$$

where I denote identity matrices of dimension 3 on 3, and matrices H_{iE}, $\Omega u \dot{\Omega}$ are of the kind given above. Thus, transformation (32) in which the matrix Ξ_{iE} looks like (37), together with the equations (31)–(36), solves the problem of transformation of coordinates, velocities and accelerations from the Greenwich system into an inertial system.

If the equations of the inverse algorithm are used, it is enough to transpose separately each of the matrices making up the hypermatrix Ξ_{iE}.

It is of importance and of interest to obtain an estimation of the accuracy of the equations developed here.

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