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Robustification of Tangent Angle Function Snakes

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Abstract: The internal energy E_{int} of conventional snakes contains the first and second derivatives of the coordinates with respect to arc length. Therefore, the Eulerian equations equivalent to the variation problem are of the fourth order, and the coefficient matrices of the linear equation systems belonging to it are pentadiagonal. If E_{int} is parameterized with respect to the curve direction and curvature, the Eulerian equation of these so-called tangent angle function snakes is now only of the second order, and the coefficient matrix of the linear system is tridiagonal. This comparatively simple snakes algorithm works quickly and reliably on smooth curves. Used on the cartographic displacement of line objects, shape distortions show up on strongly curved and non-equidistant digitized lines. The causes for the defects were examined, eliminated, and, so, the procedure robustified.

Zusammenfassung: *Robustifizierung der Tangent Angle Function Snakes.* Die innere Energie E_{int} konventioneller Snakes enthält die ersten und zweiten Ableitungen der Koordinaten nach der Bogenlänge. Deshalb sind die zum Variationsproblem äquivalenten Euler-Gleichungen von vierter Ordnung und die Koeffizientenmatrizen der zugehörigen linearen Gleichungssysteme pentadiagonal. Parametrisiert man E_{int} nach der Kurvenrichtung und -krümmung, so ist die Euler-Gleichung dieser so genannten Tangent Angle Function Snakes nur noch von zweiter Ordnung und die Koeffizientenmatrix des linearen Systems tridiagonal. Dieser vergleichsweise einfache Snakes-Algorithmus arbeitet schnell und zuverlässig an glatten Kurven. Angewendet auf die kartographische Verdrängung von Linienobjekten zeigten sich Formstörungen an stark gekrümmten und ungleichabständig digitalisierten Linien. Die Ursachen der Defekte wurden untersucht, beseitigt und damit das Verfahren robustifiziert.

1 Introduction

As well known, the snakes technology is derived from the energy minimum principle. Since the latter is a universal one, also alternative tasks can be solved with snakes (MEIER 2000), aside from recognition and extraction problems of digital image processing like, for example the displacement of line objects in digital cartography and in terminal outputs of semantic information of any kind (BURGHARDT & MEIER 1997). Hereby, the internal energy (shape energy) with the elasticity and the stiffness term is used to retain the original shape of the lines as much as possible. In these terms, typical curve characteristics like direction and curvature are contained implicitly.

The shape-retaining line displacement suggests parameterizing the shape energy directly in respect to curve direction and curvature. From this, a new species of snakes is derived with only one Eulerian equation of, yet, a different kind and, after its discretization, also a differently structured equation system with considerable consequences for the numerical calculation (paragraph 2). The new snakes algorithm developed by BORKOWSKI et al. (1999) is somewhat faster than the original one created by KASS et al. (1987) and delivers approximately the same stable solutions to sufficiently smooth and dense enough digitized lines. Nevertheless, undesirable shape distortions up to breaks/corners were found on strongly curved, nonequidistant digitized lines from time to time during tests with real data. The causes for the instabilities were examined, eliminated and, so, the algorithm robustified (paragraph 3).

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2 Basic Relations of Tangent Angle Function Snakes

In the conventional snakes model (KASS et al. 1987), compare to Tab. 1, the internal energy E_{int} from the elasticity term and the stiffness term are combined in linear fashion. The prior contains the first derivatives, the latter the second derivatives of the planar coordinates $x = x(s)$, $y = y(s)$ in respect to the arc length s. The parameters α and β weigh down both terms respectively, as well as E_{int} against the external energy E_{ext}. They can be constant or position-dependent. Furthermore, they can be changed (interactive controlling). The Eulerian equations in Tab. 1 are noted down for constant α and β. As they are of the fourth order in x, y, five points are needed in the discrete approximation. The coefficient matrix $\boldsymbol{A}_P$ of the linear equation systems with respect to the x, y-coordinates is pentadiagonal. If E_{int} from the curve direction $\varphi(s)$ and its first derivative $\dot{\varphi}(s)$, identically with the curvature, is combined, the Eulerian equation comes about in $\varphi(s)$. Therefore, we have called the new species of snakes described in detail by BORKOWSKI et al. (1999) **T**angent **A**ngle **FU**nction **S**nakes (TAFUS). Their main characteristics, when compared to conventional snakes, are the following (compare Tab. 1):

In lieu of *two* Eulerian equations of the fourth order in $x(s)$, $y(s)$, there is, now, *one* equation of the second order in $\varphi(s)$, thus,

Tab. 1: Basic relations of Tangent Angle Function Snakes (left) compared with those of conventional snakes (right). The points denote derivations with respect to the arc length s. For details see chapter 2.

Tangent Angle Function Snakes	Conventional Snakes
Internal energy (shape energy)	
$\alpha\varphi^2 + \beta\dot{\varphi}^2$ direction curvature $\varphi(s) = \arctan(\dot{y}/\dot{x})$ $\dot{\varphi}(s) = \dot{x}\ddot{y} - \dot{y}\ddot{x}$	$\alpha\lvert\dot{\underline{v}}\rvert^2 + \beta\lvert\ddot{\underline{v}}\rvert^2$ elasticity stiffness $\dot{\underline{v}} = \begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix}, \ddot{\underline{v}} = \begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix}$
External energy (conflict energy) $E_{ext} = \begin{cases} 1 - a/h & : a < h \\ 0 & : a \geq h \end{cases}$	
$-\dfrac{\partial E_{ext}}{\partial \varphi} = \dfrac{1}{h}\dfrac{\partial a}{\partial s}\dfrac{\partial s}{\partial \varphi} = \dfrac{1}{h}\dfrac{\partial a}{\partial s}\dfrac{1}{\dot{\varphi}}$	$-\dfrac{\partial E_{ext}}{\partial x} = \dfrac{1}{h}\dfrac{\partial a}{\partial s}\dfrac{\partial s}{\partial x} = \dfrac{1}{h}\dfrac{\partial a}{\partial s}\cos\varphi$ $-\dfrac{\partial E_{ext}}{\partial y} = \dfrac{1}{h}\dfrac{\partial a}{\partial s}\dfrac{\partial s}{\partial y} = \dfrac{1}{h}\dfrac{\partial a}{\partial s}\sin\varphi$
Eulerian equations	
$\dfrac{\partial E_{ext}}{\partial \varphi} - \alpha\varphi + \beta\ddot{\varphi} = 0$ + Restriction: Snakes points move vertically to the snakes direction	$\dfrac{\partial E_{ext}}{\partial x} - \alpha\ddot{x} + \beta x^{(4)} = 0$ $\dfrac{\partial E_{ext}}{\partial y} - \alpha\ddot{y} + \beta y^{(4)} = 0$
Discretization (FDM)	
3-direction-approximation $\underline{A}_T\underline{\varphi} = \underline{b}_\varphi$ $\underline{A}_T$ tridiagonal	5-point-approximation $\underline{A}_P\underline{x} = \underline{b}_x$ $\underline{A}_P\underline{y} = \underline{b}_y$ $\underline{A}_P$ pentadiagonal

discretely, only *one* equation system to be solved. The coefficient matrix $\boldsymbol{A}_P$ is tridiagonal with a very noticably better condition than $\boldsymbol{A}_P$. The TAFUS-solution, however, is not unique: the new side directions of the deformed polygon snakes are obtained, yet not the new side lengths. In order to be able to calculate the coordinates of the deformed snakes an additional condition is called for: of course, we determined that the snakes points move vertically to the snakes direction. These transversal displacements can be calculated at small changes in direction $\delta\varphi$ by way of the arc formula and lastly, from this, the coordinates. Otherwise, the coordinates are determined by means of an intersection (a section of two straight lines noted in point-direction equations); compare Fig. 1 and Tab. 2.

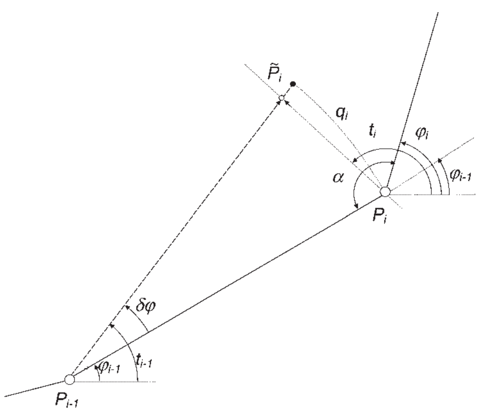

Fig. 1: Displacement by the TAFUS algorithm. The polygon snake direction φ_{i-1} turns into the new direction $\varphi_{i-1} + \delta\varphi$. For computation of coordinates of the displaced point $\tilde{P}_i$ see Tab. 2.

The term E_{ext} is shown in Tab. 1 as conflict energy in the case of (cartographic) line object displacement: $E_{ext} > 0$ if the distance of neighboring line objects $a = a(s)$ is smaller than a given hardcore distance $h = const$, $E_{ext} = 0$, if no conflict exists with $a \geq h$. The derivatives of E_{ext} with respect to x, y (snakes) or with respect to φ(TAFUS), which constitute the inhomogenities of the linear equation systems in both models, aside from the hardcore distance h, contain conflict change $\partial a/\partial s$ along s and in the snake-model additionally the direction φ as well as the curvature $\dot{\varphi}$ in the TAFUS-model. For both, the different coefficient matrices ($\boldsymbol{A}_T$, $\boldsymbol{A}_P$) as well as the inhomogenities ($\boldsymbol{b}_x$, $\boldsymbol{b}_y$; $\boldsymbol{b}_\varphi$) TAFUS must be controlled differently from conventional snakes for obtaining equivalent results.

3 Robustification of Tangent Angle Function Snakes

The TAFUS-algorithm works dependably on sufficiently smooth, equidistant and densely enough discretized curves. In tests with real data shape distortions were obtained when the curves were highly curved, had

Tab. 2: Computation of the TAFUS coordinates $\tilde{x}_i$, $\tilde{y}_i$ by the arc formula (left) or by intersection (right); see also Fig. 1. Deviations between the results of both methods are outlined in chapter 3, ii).

Arc formula	Intersection
$\tilde{x}_i = x_i + \delta x_i$ $\tilde{y}_i = y_i + \delta y_i$	
$\delta x_i = q_i \cos\left(\varphi_{i-1} + \frac{\pi}{2}\right)$	$\delta x_i = \frac{y_i - y_{i-1} + (x_i - x_{i-1}) \tan t_{i-1}}{\tan t_{i-1} - \tan t_i}$
$\delta y_i = q_i \sin\left(\varphi_{i-1} + \frac{\pi}{2}\right)$	$\delta y_i = (\tilde{x}_i - x_i) \tan t_i$
$q_i = s_i \cdot \delta\varphi_{i-1,i}$	$t_{i-1} = \varphi_{i-1} + \delta\varphi_{i-1,i}$
$s_i^2 = (x_i - x_{i-1})^2 + (y_i - y_{i-1})^2$	$t_i = \frac{1}{2}(\pi + \varphi_{i-1} + \varphi_i)$ if $\delta\varphi > 0$
	$t_i = \frac{1}{2}(3\pi + \varphi_{i-1} + \varphi_i)$ if $\delta\varphi < 0$

breaks/corners and/or had been locally variably digitized. In order to robustify the procedure, the possible causes for defects were sought:

i) *Definition of the conflict energy* (see Tab. 1).
According to the definition E_{ext} diminishes from the position of the maximum ($a = 0$) to the position of the just eliminated conflict ($a = h$) in linear fashion. If E_{ext} is set up as exponentially decreasing in a, the snake at first comes close in an iteratively fast way but then in a slower one to the position sought. Our tests have shown that the quality of the solution is independent of a special monotonely decreasing function type for E_{ext}.

ii) *Calculation of the coordinates* (see Tab. 2).
In the iterative algorithm, the direction changes $\delta\varphi$ and the coordinates changes δx_i, δy_i are small quantities. Therefore, the new coordinates $\tilde{x}_i$, $\tilde{y}_i$ of snake points P_i had, up to now, been approximated with the arc formula. This solution was compared to the correct one: Intersecting, starting from points P_{i-1}, P_i with directions t_{i-1}, t_i (see Fig. 1 and Tab. 2).

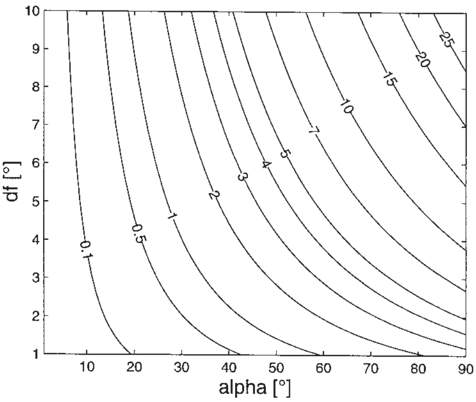

Fig. 2: Deviation of position (in percent) computed by two variants according to Tab. 2, related to the curvature radius r, and as function of the break angle α (in degree), and of the change df$\equiv\delta\varphi$ of the polygon side direction φ (in degree). For discussion see chapter 3, ii).

The position error of the approximated solution depends on $\delta\varphi$, α and the local curvature radius r of the curve. In Fig. 2 the lines of the equal position deviation are shown in respect to r; for example, if $\delta\varphi = 4^\circ$, $\alpha = 90^\circ$ a deviation of 10% is obtained, thus 1 mm for $r = 10$ mm. On real line objects, the order of the magnitude of one percent is, as a rule, hardly ever surpassed. Therefore, the undesirable defects could be eliminated by intersecting to only a small degree (not completely).

iii) *Spacing of digitized data.*
Irregular spacing has shown itself to be the main source for instabilities. In the conventional snakes-algorithm, deviations from the equidistance, with the exception of long pieces of straight lines (without interpolated points), hardly play a role (Burghardt & Meier 1997). They are, however, of importance to the TAFUS-algorithm.

The necessity for interpolating the data at first seems to be a disadvantage, rather causing the reduction of the TAFUS potencies. But this is only partially so, for also in the conventional snakes-algorithm there is interpolation and this even in *every* iteration step for finding out the respective conflict energy E_{ext}. Fig. 3 shows that this is necessary: E_{ext} is calculated from the discrete distances a_i between the points of closely neighboring pieces of curves: $E_{ext} > 0$, expressed by distances a_i, which are smaller than a given hardcore distance h (Fig. 3, right). If, however, polygon side lengths surpass the value $2h$, then $a_i < h$ and $E_{ext} = 0$ are possible (Fig. 3, left) although a conflict exists. In this case, not only E_{ext} but also $\partial E_{ext}/\partial\varphi \sim \partial a/\partial s$, meaning the change of conflict along the curve(s) and, thus, the inhomogenity of the linear equation system (see Tab. 1), is calculated incorrectly in a local neighborhood of the critical spots, especially when small and large point distances alternate. The defects named can be eliminated by a *onetime* approximated equidistant interpolation of the input data. However, one needs to, then, work with more extented data sets.

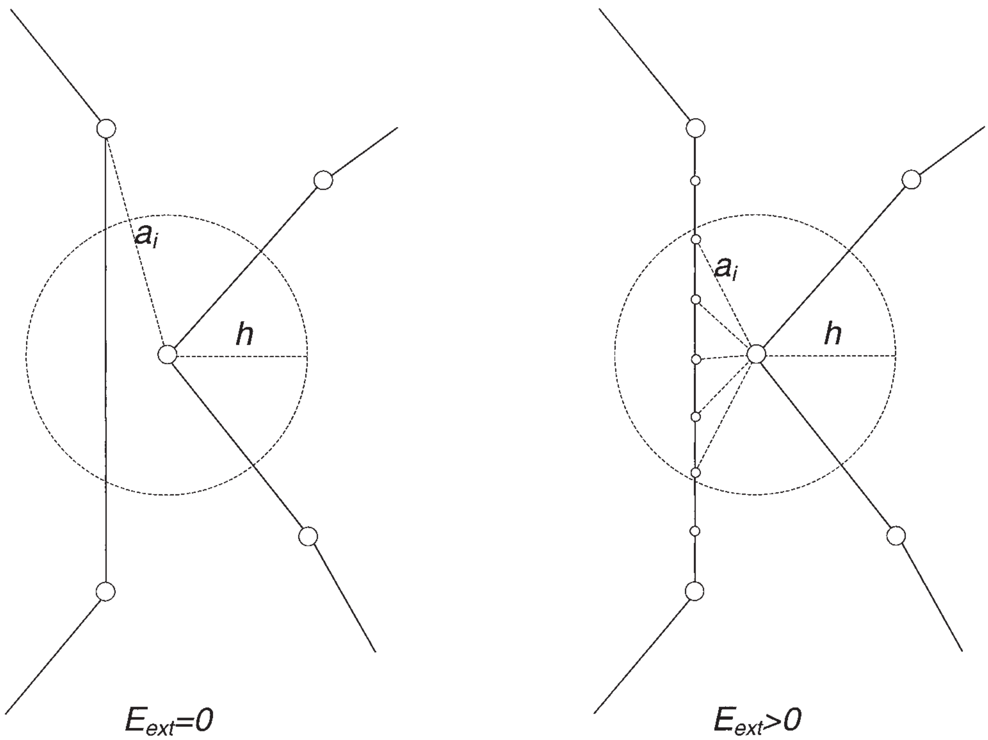

Fig. 3: Numerical computation of the external energy E_{ext} from interpolated data. For discussion see chapter 3, iii).

The situation is shown in the test example in Fig. 4. The line objects, which partially touch or even cover each other (a), were not equidistantly digitized (b). The displacement solution with the TAFUS-algorithm (c) shows shape distortions; especially breaks were increased. With the interpolated data shape-stable solutions are obtained by means of the arc formula (e) as well as by intersecting (f).

4 Conclusion

The interpolated data in Fig. 4 (d) so-to-speak represent a vectorial pendant to regular raster data, yet with directions $\varphi_i \in (0,2\,\pi)$. Therefore, in the TAFUS-algorithm with vector data also small, especially arbitrary small changes $\delta\varphi_i$ are possible; yet in the grid with an 8-neighborhood only discrete direction breaks of about at least $\pi/4$. Thus, the TAFUS-algorithm is limited to special applications. The same goes also for several other snakes models created in the digital image processing like ribbons, twins, ziplocks, and others. The TAFUS-model is no exception in this respect.

In the transition from the continuous Eulerian equations to the linear equation systems, the equidistant discretization is, of course, supposed. Yet, irregularly digitized line objects occur rather often in practice, for example in the geo-data storage ATKIS used in Germany. Hereby, the TAFUS-algorithm reacts more sensitively than conventional snakes. Instabilities regarding the curve shape can, however, be avoided by using approximative equidistant data.

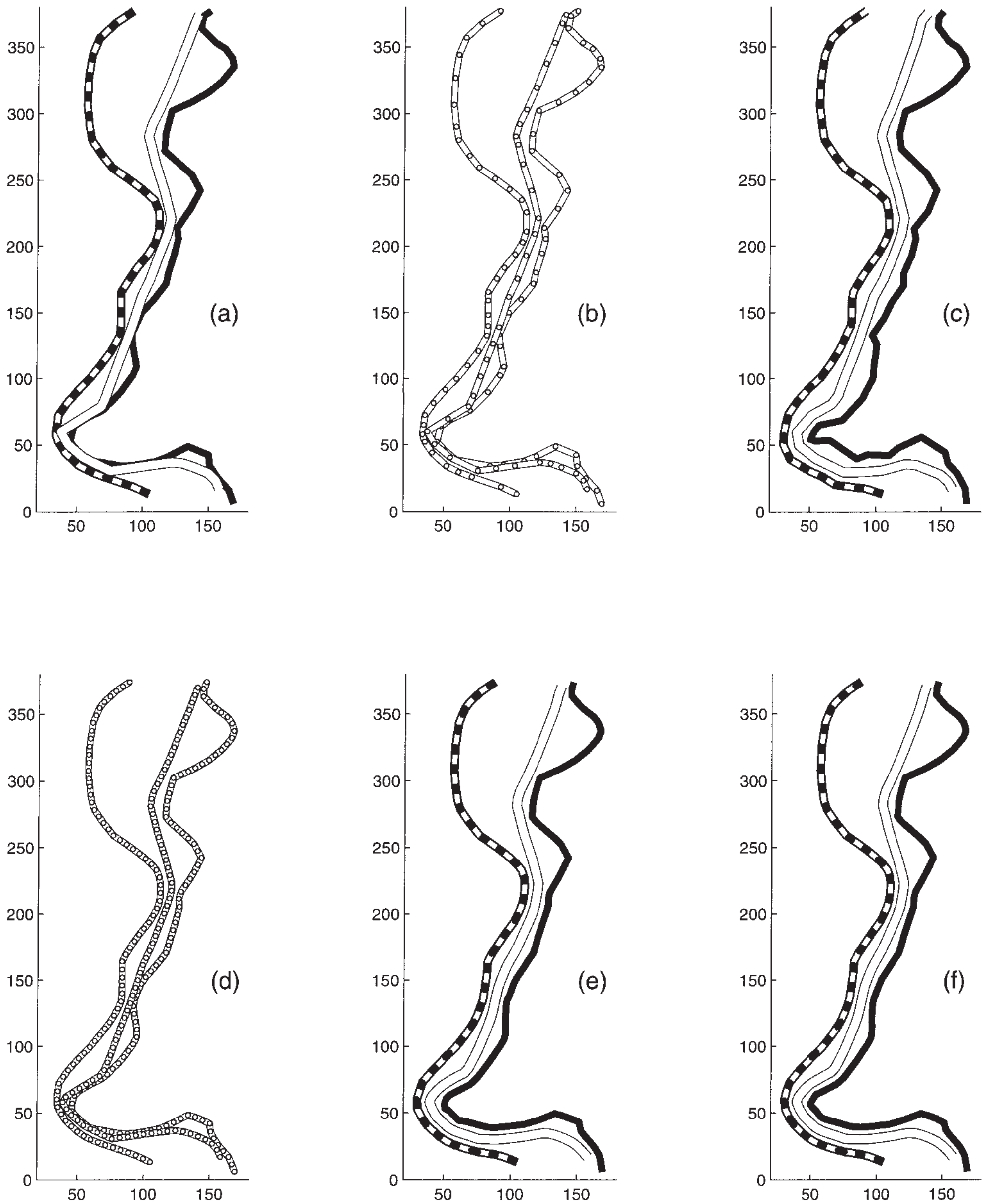

Fig. 4: Test example. Variants of line objects displacement by the TAFUS algorithm with non-regular and regularized data, discussed in chapter 3, iii).

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